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Fractal Predictor of the Dynamics of Bitcoin Fluctuations by Using Self-Affine Time Series

Jaime Moreno[1], Oswaldo Morales[1], Ana Lilia Coria[2], Ricardo Tejeida[3], Liliana Chanona[1]

[1] Instituto Politécnico Nacional,
Escuela Superior de Ingeniería Mecánica y Eléctrica,
México

[2] Instituto Politécnico Nacional,
Escuela Superior de Comercio y Administración,
México

[3] Instituto Politécnico Nacional,
Escuela Superior de Turismo,
México

jemoreno@esimez.mx

Abstract. The use of cryptocurrency has boomed in recent years, such as Bitcoin, Ethereum or Ripple. It is interesting to have a Bitcoin forecasting tool to try to understand the trends at the global economic level. A virtual currency that can be used as a means of payment just like physical money. Any cryptocurrency uses peer-to-peer technology and is not controlled by any economic or political entity, such as a bank or government. In the year of 2009, Bitcoin was conceived and was priced at 0.39 USD reaching its all-time high in 2017 with a price of 17,549.67 USD, i.e. 45 thousand times more in less than 10 years. This work focuses on predicting the bitcoin-price trending will have in the year 2020 by using a Self-Affine Fractal Analysis as a tool of artificial intelligence. The results that the present work provides in the first 6 months agree with 98% with those actually obtained despite training only with the data from the first days of time series.

Keywords: Fractal predictor, Bitcoin, cryptocurrencies, self-affine analysis, fluctuations.

1 Introduction

Cryptocurrencies or virtual currencies are offered through the internet globally and are sometimes presented as an alternative to legal tender, although it has very different characteristics:

– It is not mandatory to accept them as a means of paying debts or other obligations.

– Its circulation is very limited.
– Its value fluctuates strongly, so it cannot be considered a good store of value or a stable unit of account.

The strong fluctuations experienced by these cryptocurrencies that seem typical of the classic speculative bubbles are well known. As an example, the average value of bitcoin on the main platforms in which it is traded increased in 2017 from approximately 979.53 USD per unit at the beginning of the year to more than 17,147.04 euros in mid-December. Since then, the trend has been downward. As of February 5, 2018, its price was below 6,902.35 USD, which represents a drop of more than 65% from the December highs. A person who had bought bitcoins in late 2017 and sold them today would suffer very noticeable losses [20].

Additionally, numerous fundraising actions are taking place from investors to finance projects through the so-called initial offers of cryptocurrencies or ICOs. The expression ICO can refer to both the actual issuance of cryptocurrencies and the issue of rights of various kinds, generally called tokens. These assets are put up for sale in exchange for cryptocurrencies such as bitcoins or ethers or official currency (for example, US dollars or USD)[12].

The five main aspects to consider before investing in cryptocurrencies or participating in an ICO:

1. To date, no ICO has been registered, authorized or verified by any supervisory body in Spain. Therefore, there are no cryptocurrencies or tokens issued in ICO whose acquisition or possession worldwide can benefit from any of the guarantees or protections provided in the regulations regarding banking or investment products. Investments in cryptocurrencies or in ICOs outside the regulation are not protected by any mechanism similar to that which protects cash or securities deposited in credit institutions and investment services companies.
2. Before buying this type of digital assets or investing in products related to them, consider all the associated risks and assess whether you have enough information to understand what is being offered. In this type of investment there is a high risk of loss or fraud.
3. Cryptocurrencies lack intrinsic value, becoming highly speculative investments. Furthermore, its strong dependence on poorly consolidated technologies does not exclude the possibility of operational failures and cyber threats that could mean temporary unavailability or, in extreme cases, the total loss of the amounts invested.
4. The absence of markets comparable to organized securities markets subject to regulation can hinder the sale of cryptocurrencies or tokens issued in ICO to obtain conventional cash.
5. In the case of ICOs, the information made available to investors is usually not audited and is often incomplete. Generally, it emphasizes potential benefits, minimizing references to risks.

2 Related Work

In recent years a new type of tradable assets appeared, generically known as cryptocurrencies. Among them, the most widespread is Bitcoin. Bariviera et al in [6] compared Bitcoin and standard currencies dynamics, analyzing their returns at different time scales. They investigated the long memory in return time series from 2011 to 2017, using transaction data from one Bitcoin platform. In addition, they computed the Hurst exponent by means of the Detrended Fluctuation Analysis method, measuring long range dependence. They found changes in the Hurst eponent values over the first years of the studied period, tending to stabilize in the last part of the the period. In the aftermath they claim that the Bitcoin market can be described as a self-similar process, displaying persistent beavior until 2014.

Caporale et al in [7] studied the persistence behavior in the cryptocurrency market, applying the R/S analysis and fractional integration long-memory methods and taking as inputs the four main cryptocurrencies (BitCoin, LiteCoin, Ripple, Dash) over the sample period 2013–2017. According to their outcomes, they found that the cryptocurrency market displays persistence or positive correlations between its past and future values. Hence, they claim for a market inefficiency because they did not found random walk behavior (market efficiency) in the cryptocurrency market.

Owing ryptocurrencies have acquired a great development and valorization, Costa et al in [9] analyzed the following four cryptocurrencies, based on their market capitalization and data availability: Bitcoin, Ethereum, Ripple, and Litecoin, using detrended fluctuation analysis and detrended cross-correlation analysis and the respective correlation coefficient. Bitcoin and Ripple seemed to behave as efficient financial assets, while Ethereum and Litecoin displayed some persistence. When authors correlated Bitcoin with the other three cryptocurrencies, at short time scales all the cryptocurrencies had been correlated with Bitcoin, although Ripple had the highest correlations. On the other hand, at higher time scales, Ripple was the only cryptocurrency with significant correlation.

Quintino et al in [19] determined the persistence exhibited by the Bitcoin measured by the Hurst exponent from the Brazilian market daily prices from 9 April 2017 to 30 June 2018, and comparing them with Bitcoin in USD. They used Detrended Fluctuation Analysis, for the period. The analyzed the prices of Bitcoins yielded from negotiations made by two Brazilian financial institutions: Foxbit and Mercado. The authors found that Mercado and Foxbit returns followed Bitcoin dynamics and showed persistent behavior, although the persistence was higher for the Brazilian Bitcoin.

On the other hand, Haque et al in [18] studied forecasts of Bitcoin price using the autoregressive integrated moving average (ARIMA) and neural network autoregression (NNAR) models. The authors forecasted next-day Bitcoin price both with and without re-estimation of the forecast model for each step by using the static forecast approach. For cross-validation of forecast outcomes, they took into account a first training-sample where NNAR performed better than ARIMA, while ARIMA outperformed NNAR in the second training-sample. Moreover, ARIMA with model re-estimation at each step outperformed NNAR in the two test-sample forecast periods. Hence, they claim the superiority of forecast results of ARIMA model over NNAR in the test-sample periods, and therefore the ARIMA superiority of volatile Bitcoin price prediction.

Deniz and Stengos in [11] analyzed the behaviour of Bitcoin returns and those of several other cryptocurrencies in the pre and post period of the introduction of the Bitcoin futures market, using the principal component-guided sparse regression (PC-LASSO) model. The authors observed the Search intensity as the most important variable for Bitcoin for all periods, whereas for the other cryptocurrencies there were other variables that seemed more important in the pre period, while search intensity still stood out in the post period Furthermore, GARCH analyses suggested that search intensity increased the volatility of Bitcoin returns more in the post period than it does in the pre period. Therefore, the authors asserted that the top five cryptocurrencies were substitutes before the launch of Bitcoin futures.

As we observed, the Bitcoin price-returns dynamics has been characterized applying fractal analysis tools such as the Detrended Fluctuation Analysis and R/S analysis. On the other hand, the Bitcoin price has also has been forecasted by applying econometrics (GARCH) and artificial intelligence (neural network autoregression) tools. In this work, we chacracterized the Bitcoin price fluctuations by calculating the Hurst exponent (H) by estimating the structure function of the time series of Bitcoin price fluctuations in different time interval of the sample. Aditionally, we forecasted the Bitcoin prices applying an artificial intelligence tool. Hence the following sections of this work are the following. In Section 3 the Theoretical Framework is exposed with main features of BItcoin and Fractal Theory. while in Section 4, main Proposal is defined along with methodology as a set of ordered steps. Finally, In Section 5 shows the experimentation methodology as well as the most important results that prove the value of this work.

3 Theoretical Framework

3.1 BitCoin

Bitcoin is a virtual, independent and decentralized currency, since it is not controlled by any State, financial institution, bank or company. It is an intangible currency, although it can be used as a means of payment just like physical money. Virtual currencies constitute a heterogeneous set of innovative payment instruments that, by definition, lack a physical support to back them up.

The term bitcoin has its origin in 2009, when it was created by Satoshi Nakamoto (pseudonym of its author or authors) [13], who created it with the aim of being used to make purchases only through the Internet. Bitcoin was born with high ambitions: to provide citizens with a means of payment that enables the execution of fast, low-cost transfers of value and which, in addition, does not can be controlled or manipulated by governments, central banks or financial entities.

Virtual currency uses cryptography to control its creation. The system is programmed to generate a fixed number of bitcoins per unit of time through computers called miners. Currently, that number is fixed at 25 bitcoins every ten minutes, although it is programmed so that it is halved every 4 years. Thus, starting in 2017, 12.55 bitcoins will be issued every ten minutes. Production will continue until 2140, when the limit of 21 million units in circulation is reached [12].

To make use of this virtual currency, it will need to download software to the computer or our mobile that will serve as a *virtual wallet* and will generate a bitcoin address, which can be used to send and receive money from other users. In addition, the sending of bitcoins is instantaneous and all operations can be monitored in real time. Transactions with this currency involve a transfer of value between two bitcoin addresses, public although anonymous. To guarantee security, transactions are secured using a series of key cryptographies, since each account has a public and a private key. As in other virtual currencies, bitcoin also has a number of risks that must be highlighted to know exactly the magnitude of this currency. To identify them, we again resorted to the report of the General Directorate of Operations, Markets and Payment Systems of the Payment Systems Department of the Wold Bank, which groups them into:

- Financing of illegal activities and/or money laundering. Due to the decentralized nature of the scheme, transfers take place directly between the payer and the beneficiary, without the need for an intermediary or administrator. This implies a difficulty in identifying and early warning of possible suspicious behavior of illegal activities.
- The fact that organized crime networks are making widespread use of emerging electronic payment systems can create a negative reputation for digital payment methods.
- Despite the fact that, in principle, any computer can actively participate in the process of creating new bitcoin units, the high computational capacity required implies that, in practice, this activity is dominated by a small group of actors. Possible fraudulent transactions. To the extent that the protocols on which bitcoin is based are open software developments, the implementation of its different versions does not have to occur uniformly among all users.
- Impact on price stability and financial stability, since private trading platforms where Bitcoins can be exchanged for legal tender currencies are marked by the high volatility of prices due to speculative movements.
- From the point of view of fraud, bitcoin presents a significant weakness compared to other payment methods in the online world, such as cards.

3.2 Fractal Theory

Fractals are mathematical objects that generalize Euclidean geometric objects to non-whole dimensions and allow us to delve into the study of complex sys-

tems, disorder and chaos [15]. Fractals refer to any class of phenomena that possess scaling that exhibits dilatation symmetry, or scale invariance, often characterized by the appearance of a power-law. Invariant scale systems are usually characterized by non-integer dimensions. Benoit Mandelbrot developed the fractal geometry to unify a number of previous studies on irregular shapes and natural processes. Hence, fractal geometry is a mathematical tool for dealing with complex systems that do not have a characteristic scale of length, or scale invariance [15].

Mandelbrot also focused on a particular set of such objects and forms where a part of the object is identical to a larger piece, i.e. self-similar objects. According to [15], there are deterministic fractals when a small piece of a fractal is separated and isotropically magnified to the size of the original, both of them look the same. By magnifying isotropically, all the directions have been rescaled by the same factor.

On the other hand, there are systems that are invariant only under *anisotropic* magnifications, which are called *elf-affine fractals*s. If a self-affine curve or time series is invariant in scale under the transformation $x \to bx$, $y \to ay$, it is observed in Equation 1:

$$F\left(bx\right) = aF\left(x\right) \equiv b^{H} F\left(x\right), \tag{1}$$

where $H = \frac{\log a}{\log b}$ is the exponent of Hurst [15].

Time series are sets of data or records of any observable variable under study. These records are separated by a same time interval, such as seconds, minutes, hours, weeks, months, years, etc. Time series reflect the behavior of a complex system over time.

Kantz and Schreiber [14] proposed an approach to study such systems from their time series fluctuations, in order to characterize their dynamics by means of scaling laws, which are valid over a wide range of time scales and that they are a property of fractals. When carrying out a fractal characterization of time series fluctuations generated by some complex system, what is sought is to find persistent behavior, since this will allow to make probabilistic predictions about the future states of the system based on the value of the scaling exponents obtained for this behavior.

After calculating the H or roughness exponent from Equation 1, the values $H < 1/2$ indicate long-term anti-correlation (or anti-persistence) behavior: if the values of the observable variable increase, most likely the next value is less than the last, and vice versa; values $H > 1/2$ indicate positive (or persistent) long-term correlated behavior: if the values of the observable variable increase, it is most likely that the next value is greater than the last, and vice versa. Finally, for values of $H = 1/2$ do not exist correlations, i.e. a totally random behavior.

4 Proposal

4.1 Theoretical Definition

Statistically, the fluctuation or volatility of financial time series $p(t)$ are characterized by their standard deviations $v(t,\tau)$ for a sampling time interval τ considered [8], exhibiting power law correlations, so these complex systems may not respond immediately to a quantity of information that flows towards them, but react gradually in a certain period of time [14,2,4,8]. The analysis of scaling or fractal properties of fluctuations has offered relevant information about the underlying processes responsible for the observed macroscopic behavior of complex systems [1,2,4,5,8].

Moreover, in this paper is studied the long-term correlations displayed by the time series fluctuations by applying their structure function, defined by Equation 2 as follows:

$$\sigma(\tau,\delta_t) = \overline{\left\langle [\nu(t+\delta_t,\tau) - \nu(t,\tau)]^2 \right\rangle}^{\frac{1}{2}}, \tag{2}$$

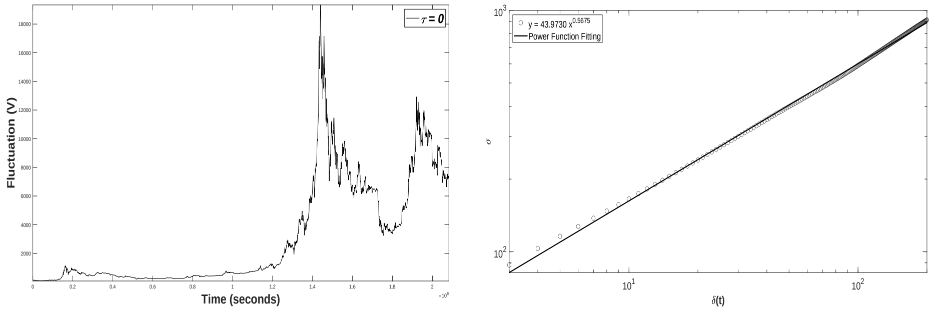
where the upper bar denotes average over all times t in the time series of length $T - \tau$ with T as the length of the original time series $p(t)$ and triangular parentheses denote average over different realizations of the time window of size δ_t [14]. The structure function of the fluctuations exhibits the power law behavior $\sigma \propto (\delta_t)^H$ with H as the local or roughness exponent, even though the time series fluctuations $\nu(t,\tau)$ exhibit apparently randomness [14,2,4,8]. The scaling behavior $\sigma \propto (\delta_t)^H$ characterizes the correlations in the time series fluctuations treated as a growing interface in a dimension $(1+1)$ [17,10,21].

Accordingly in this paper the structure function was applied to study the dynamics of time series fluctuations associated with the Bitcoin price, by analyzing the behavior of standard deviations for different sampling time intervals. Hence, the time series of standard deviations were treated as interfaces in motion, where the considered sampling time interval τ plays the role of time variable and the physical time t plays the role of the spatial variable [3].

4.2 Methodology

Within artificial intelligence there are systems that think rationally, these try to emulate the logical logical thinking of humans, that is, it investigates how to make machines perceive, reason and act accordingly. The Proposed system tries to reason the average fluctuation to propose future fluctuations and therefore the estimation of the price trend [22].

To study the dynamics of time series fluctuations, in this paper the time series of standard deviations $\nu(t,\tau)$ of the original series $p(t)$ from the open price (op) and close price (cp) of Bitcoin. For this study the length of each original financial series was $T_{op} = 2410$ and $T_{cp} = 2410$ (usd-dollar versus time). In addition, the sampling rate $\triangle t = 1$ business day, with ranges $\tau_m \leq \tau \leq \tau_m$ and rates (δ_t) from samples of time intervals (δ_t) from 3 to 200 standard deviations.

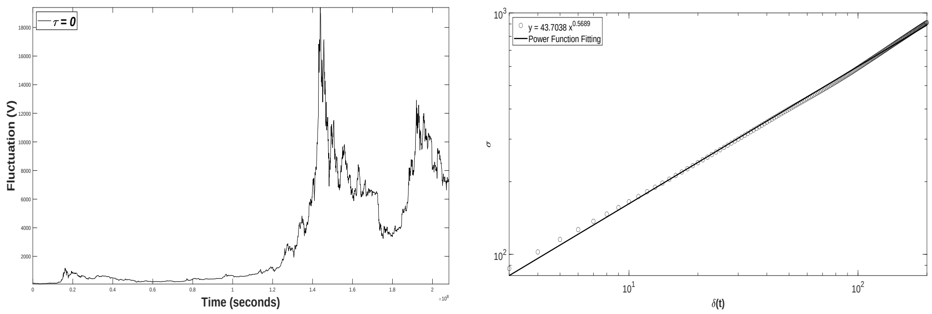


(a) Open (b) Open Fluctuation

Fig. 1. (**a**) Original time series of Bitcoin open-price, $p(t)$ (USD), with $T_{op} = 2410$. (**b**) Dynamic scaling of the structure function for the volatility time series with $H_{op} = 0.5675$.

In Figure 1(a) is shown the graph of the original time series, $p(t)$ to the Bitcoin open-price, and in Figure 1(b) is shown the structure function of the the time series of standard deviations $\nu(t, \tau)$ with a value or the Hurst exponent (or slope) equals $H_{op} = 0.5675$.

On the other hand, in Figure 2(a) is shown the graph of the original time series, $p(t)$ to the Bitcoin close-price, and in Figure 2(b) is shown the structure function of the the time series of standard deviations $\nu(t, \tau)$ with a value or the Hurst exponent (or slope) equals $H_{cp} = 0.5689$.



(a) Close (b) Close Fluctuation

Fig. 2. (**a**) Original time series of Bitcoin close-price, $p(t)$ (USD), with $T_{op} = 2410$. (**b**) Dynamic scaling of the structure function for the volatility time series with $H_{cp} = 0.5689$.

Algorithm to determine fractal behavior of the volatility time series is the following steps:

1. Collect time series with at least 2000 data of past tendency about Bitcoin open-price and Bitcoin close-price and train the system with the H findings found.

2. Construct one hundred ninety-eight time series of standard deviations (fluctuations or volatility) for every original time series, applying the equation of the standard deviation, for different sample intervals: $3 \leq \tau \leq 200$. Then, construct 198 time series $\nu(t, \tau)$ for both samples.
3. Determine the type of correlations displayed by the time series of fluctuations $\nu(t, \tau)$, applying the Equation 2 of the structure function for predicting the future tendencies.
4. If the fluctuation structure function, obtained in Step 3, exhibits a power law behavior $\sigma \propto (\delta_t)^H$, then, from the same equation, obtain the dynamic exponent of roughness or local H, in order to determine if the system displays positive correlations (persistence) in the long-term and to establish at what time the fluctuations move from a power-law behavior to one of saturation (horizontal curve or zero slope). This is done with the purpose of establishing if the behavior of the Bitcoin price fluctuations over the time can be described in the Family-Vicsek fractal model $w(L, t) \sim L^H f\left(\frac{t}{L^{\frac{H}{\beta}}}\right)$, which represents the dynamic scaling behavior of self-affine surfaces in motion.
5. Values of the dynamic scaling exponent H are obtained for both samples. If, based on the values of the exponents H, the fluctuation behavior is not fitted to the behavior described by the Family-Vicsek model, look for another model that explains and predicts the behavior of fluctuations for both the Bitcoin open-price and the Bitcoin close-price.
6. Finally, trend findings are projected and the future trend of Bitcoin prices is proposed through a power law in a logarithmic space.

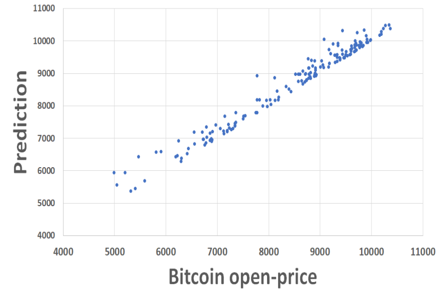
5 Experimental Results

5.1 Results

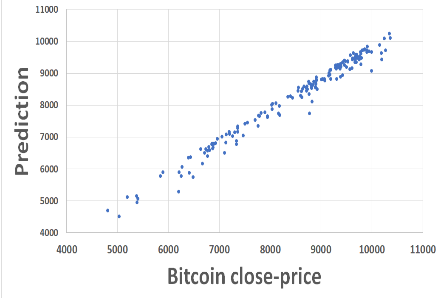
The database of the opening and closing prices of the bitcoin-USD price was downloaded from the Markets Insider site, from January 27, 2013 to June 20, 2020 [16]. Then, The original series was splitted into two parts from May 27, 2013 to December 31, 2019, with the aim of predicting the first half of 2020.

Quantitatively, the self-affinity of the time series of Bitcoin price fluctuations was characterized by the scaling behavior $\sigma \propto (\delta_t)^H$, as shown in Figures 2 and 1. The structure function displays a power law $\sigma \propto (\delta_t)^H$ with $H(\tau) = const$ within a range of intervals δ_t. In the Figures 1(b) and 2(b) the graphs of the dynamic scaling of the structure function for the Bitcoin open-price fluctuations and the Bitcoin open-price fluctuations, respectively, we observe the following values: $H_{op} = 0.5672$ and $H_{cp} = 0.5689$. It means that fluctuations for both Bitcoin open-price and Bitcoin close-price display and persistent behavior. Therefore, the dynamics of both Bitcoin prices are fitted to the power-law behavior ranging much more time scales (scale-invariance), and hence claiming a better approximation to the F-V model.

Figures 3(a) and 3(b) show the scatter plot of the actual price of the bitcoin versus the predicted one, respectively. The correlation of these data show us that



(a) Open



(b) Close

Fig. 3. Actual Bitcoin price vs Predicted Bitcoin price, $p(t)$ (USD), with $T_{op} = 172$. (**b**) Open and (**a**) Close.

the prediction of the opening of the bitcoin price has an effectiveness of 98.48%, while for the close of the trading day of 98.74%.

5.2 Discussion

According to the values of H for the Bitcoin open-price and the Bitcoin close-price, it could be pointed out the existence of a dynamic scaling behavior similar to the dynamic scaling of Family-Vicsek $(F - V)$ for the roughness kinetics of a moving interface [17], see Figures 1(b) and 2(b). The scaling relation $\sigma \propto (\delta_t)^H$ implies that the structure function of the time series of fluctuations displays the dynamic scaling behavior $\sigma(\tau, \delta_t) \propto \tau^\beta f\left(\frac{\delta_t}{\tau^{\frac{\beta}{H}}}\right)$, where the scaling function behaves like $f(u) \propto u^H$ if $u < 1$ and like $f(u) \propto$ const if $u \gg 1$. That is, the dynamic scaling relations of $F - V$, expressed in power laws, with critical scaling exponents that reflect the scale invariance of the time series fluctuations of both Bitcoin prices. Finally, it should be noted that, because the values of the scaling exponents H_{op} and H_{cp} are greater than 0.5, in the critical area the fluctuations of Bitcoin prices display positive long-term correlations, i.e. persistent behavior.

From Figure 4, the power law increment in the width transitates to a saturation regime (horizontal region) during which the width reaches a saturation value, w_{sat}. As L grows, w_{sat} increases as well, and the dependency likewise follows a power law, $w_{sat}(L) \sim L^H$ with $[t \ll t_x]$. The exponent H, *the roughness exponent*, characterizes the degree of roughness of the saturated interface.

For small u, the scaling function is increased as a power law. In this regime we have $f(u) \sim u^H$ with $[u \ll u_x]$.

As $t \to \infty$, the width saturates. Saturation is reached for $t \ll t_x$, that is $u \gg 1$. In this limit $f(u) =$ constant with $[u \gg 1]$.

The saturation time t_x, with the saturation width, w_{sat}, increases with the size of the system, which suggests that the saturation phenomenon constitutes a finite size effect. This leads us to affirm that you can predict what the fluctuations will be and therefore the opening and closing price of bitcoin with respect to the USD, our results indicate that there is a correlation greater than 98%.

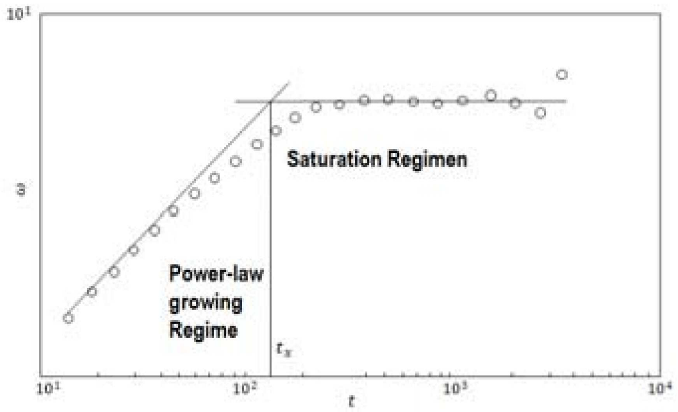


Fig. 4. Growth of the interface width time for the BD model for a horizontal-sized system [5].

6 Conclusions

The dynamic fluctuations of Bitcoin open-price and Bitcoin close-price exhibited persistent behavior. Hence it can be modeled by the family-Vicsek model.

Moreover, the dynamic scaling of $\sigma(\tau, \delta_t) \propto \tau^{\beta} f\left(\frac{\delta_t}{\tau^{\frac{\beta}{H}}}\right)$, $f(u) \propto u^H$ if $u < 1$ and $f(u) \propto$ const if $u \gg 1$ allowed to treat the dynamics of fluctuations of both Bitcoin prices as a kinetic roughening of a moving interface. The findings of this work point out to the existence of a dynamic scaling behavior similar to the dynamic scaling of Family-Vicsek for the kinetic roughening of a moving interface. Therefore, the kinetic roughening theory tools can be used to characterize and model the time series fluctuations of Bitcoin price.

This work is a novel algorithm that can predict what the fluctuations will be and therefore the opening and closing price of bitcoin with respect to the USD can be estimated. Our results indicate that there is a correlation greater than 98 %, which means that it has high reliability.

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